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Fractional Spin of System With Chern–Simons Term Coupled to Polaron

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The fractional spin of a system with Chern–Simons (CS) term coupled to a polaron at the quantum level is studied. The Faddeev–Senjanovic (FS) scheme for path-integral quantization of constrained Hamiltonian systems is applied. The quantal conserved angular momentum and the fractional spin at the quantum level of this system are presented based on the quantal Noether theorem. The fractional spin is also presented for the system with Maxwell kinetic term.

KEY WORDS: constrained Hamiltonian system; fractional spin; Chern-Simons.

1. INTRODUCTION

The concepts of fractional spin and statistics are special to two spatial dimentions (Wilczeck, 1982). They have attracted considerable attention (Kim and Lee, 1994; Kim and Park, 1994; Wilczeck and Zee, 1983), partly because of the suggestion that the quantum Hall effect and high- T_c superconductivity are planar physical phenomena that such concepts might describe correctly with CS theory. The Abel CS theory minimally coupled to the matter fields is usually considered as the base system at the field-theoretical level and the property of fractional spin and statistics is possible provided these field theories contain the CS term (Kim, Kim, and Shin, 1994). CS gauge field does not have real dynamics of its own whose dynamics comes from the field to which it is minimally coupled. In the discussion of the angular momenta for anyons, the results were deduced by using the symmetric energy-momentum tensor or the classical Noether theorem (Banerjee, 1994; Bannerjee, 1993) and it needs further study that whether they are valid at the quantum level or not. The models with CS term coupled to Scalar and spinor

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QED are discussed (Jiang and Li, 1999; Li and Li, 2002) in which the fractional spin and statistics is revealed at the quantum level. Is there still the fractional spin and statistics property in the system of Chern–Simons term coupled to polaron. We shall study this problem at the quantum level.

In the formulation of path-integral quantization, the main ingredient is the classical action together with the measure in the space of field configurations. Thus, the path integrals provide a useful tool for studying the quantal symmetries of a system. The phase-space path integrals are more fundamental than the configurations-space path integrals. In this paper, the property of fractional spin of the system with CS term coupled to polaron at the quantum level is studied. According to the rule of path integral quantization for constrained Hamiltonian system in FS scheme, the system is quantized. Based on the quantal Noether theorem, the quantal conserved quantities (including conservation of angular momentum) have been calculated, and the fractional spin at the quantum level of this system is presented. The system is presented. The system including the Maxwell kinetic term is also considered. In this case, the property of fractional spin is also presented.

2. CHERN–SIMONS TERM COUPLED TO POLARON

The electron-phonon system (polaron) is basic to the BCS theory of superconductivity for metals. The interaction has been described by using a singular Lagrangian in $(1 + 1)$-dimensional space-time and the canonical quantization for this Lagrangian was given by using Dirac brackets (Rodriguez-Nunez, 1990). But the electronic field in this Lagrangian satisfies the Schrodinger's equation which cannot reflect the correct anti-commutation relations after quantization and describe the spin of electron. Here we use Dirac's spinor to describe the electron and those difficulties above will be eliminated. Thus, the (2+1)-dimensional Lagrangian of the electron–phonon system can be modified by

$$\mathcal{L} = \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi + \frac{1}{2}[\rho\dot{q}^2 - s(\nabla q)^2] - G\overline{\psi}\gamma^0\psi q \tag{1}$$

where ρ, s, and G are parameters, ψ is the spinor field and $\overline{\psi} = \psi^{+}\gamma^0$, q is the phonon field. Throughout this paper the same notation as in Rodriguez-Nunez (1990) will be used.

The Lagrangian of the system with Chern–Simons term coupled to polaron in (2+1)-dimensional space-time is given by

$$\begin{aligned}\mathcal{L} = {}& \frac{\kappa}{4}\varepsilon^{\mu\nu\lambda}A_{\mu}\partial_{\nu}A_{\lambda} + \overline{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi + \frac{1}{2}\{\rho[(\partial_0 - iA_0)]^2 \\ & -s[\partial_i - iA_i)q]^2\} - G\overline{\psi}\gamma^0\psi q \end{aligned}\tag{2}$$

where the covariant derivative is defined by$D_{\mu} = \delta_{\mu} - iA_{\mu}$. A_{μ} is CS gauge field.

The Lagrangian density (2) is singular in the sense of Dirac. First we determine the constraints of this system in phase space. The canonical momenta conjugate to the fields A_μ, ψ, $\overline{\psi}$, and q are

$$\pi^0 = \frac{\partial \mathcal{L}}{\partial \dot{A}_0} = 0, \pi_\psi = \frac{\partial_r \mathcal{L}}{\partial \dot{\psi}} = i\overline{\psi}\gamma^0, \pi_{\overline{\psi}} = \frac{\partial_r \mathcal{L}}{\partial \dot{\psi}} = 0, \pi_q = \frac{\partial \mathcal{L}}{\partial \dot{q}} = \rho \dot{q} \tag{3a}$$

and

$$\pi^i = \frac{\partial \mathcal{L}}{\partial \dot{A}_i} = \frac{\mathsf{k}}{4}\varepsilon^{ij} A_j, \tag{3b}$$

respectively. The primary constraints of the system are given by

$$\begin{aligned} &\theta_1^0 = \pi^i - \frac{k}{4}\varepsilon^{ij}A_j \approx 0, \quad \theta_2^0 = \pi^0 \approx 0, \quad \theta_3^0 = \pi_\psi - i\overline{\psi}\gamma^0 \approx 0, \\ &\theta_4^0 = \pi_{\overline{\psi}} \approx 0, \end{aligned} \tag{4}$$

where symbol "$\approx$" means weak equality in Dirac sense. The canonical Hamiltonian density corresponding to the Lagrangian (2) is given by

$$\begin{aligned} \mathcal{H}_c &= \pi_\mu A^\mu + \pi_\psi \dot{\psi} + \pi_{\overline{\psi}} \dot{\overline{\psi}} + \pi_q \dot{q} - \mathcal{L} \\ &= \overline{\psi}(-i\gamma^i D_i + m)\psi + \frac{1}{2\rho}\pi_q^2 + \frac{s}{2}(\nabla q)^2 + G\overline{\psi}\gamma^0\psi q - \frac{1}{2}sA_i^2 q^2 \\ &\quad -isA_i q\partial_i q + \frac{k}{2}\varepsilon^{ij}A_0\partial_i A_j - \overline{\psi}\gamma^0 A_0 \psi + \frac{1}{2}\rho q^2 A_0^2 + iq\pi_q A_0 \end{aligned} \tag{5}$$

The total Hamiltonian is given by

$$H_{\mathrm{T}} = \int_v d^3x\left(\mathcal{H}_c + \lambda_1\theta_1^0 + \lambda_2\theta_2^0 + \lambda_3\theta_3^0 + \lambda_4\theta_4^0\right) \tag{6}$$

The consistency conditions $\dot{\theta}_1^0 = \{\theta_1^0\}, H_{\mathrm{T}} \approx 0$ and $\dot{\theta}_2^0 = \{{\theta_2}^0\}, H_{\mathrm{T}} \approx 0$ lead to secondary constraints

$$\theta_1^1 = sA_i q^2 + is\partial_i q + \overline{\psi}\gamma^i\psi - \frac{k}{2}\varepsilon^{ij}\partial_j A_0 \approx 0 \tag{7}$$

$$\theta_2^1 = -\frac{k}{2}\varepsilon^{ij}\partial_i A_j - \rho q^2 A_0 - iq\pi_q + \overline{\psi}\gamma^0\psi \approx 0 \tag{8}$$

The stationarity of the primary constraints $\{{\theta_3}^0, H_{\mathrm{T}} \approx 0\}$ and $\{{\theta_4}^0, H_{\mathrm{T}} \approx 0\}$ lead to the equations for determining the Lagrange multipliers λ_4, λ_3.

$$i\gamma^0\lambda_4 = \overline{\psi}(i\gamma^i D_i - m) - G\overline{\psi}\gamma^0 q + G\overline{\psi}\gamma^0 A_0 \tag{9}$$

$$i\gamma^0\lambda_3 = -(i\gamma^i D_i - m)\psi + G\gamma^0 q\psi - G\gamma^0 A_0\psi \tag{10}$$

The consistency condition of the second constraints $\dot{\theta}_1^1 \approx 0, \dot{\theta}_2^1 \approx 0$, lead to the equation for determining the Lagrange multipliers λ_2, λ_1

$$\frac{k}{2}\rho\varepsilon^{ij}\partial_j\lambda_2 = -s(2A^i q - is\partial^i)(\pi_q + i\rho q A_0) \tag{11}$$

$$\frac{k}{2}\varepsilon^{ij}\partial_j\lambda_1 = i\frac{1}{\rho}\pi_q^2 - isq\partial_i\partial^i q - iG\overline{\psi}\gamma^0\psi q - isA_i^2 q^2 + 2sA_i q\partial_i q + 2qA_0\pi_q \tag{12}$$

It is easy to check that the constraints are all second-class constraints. According to FS quantization formulation, the phase-space generating functional of Green function for the system (2) is given by (Senjanovic, 1976)

$$Z[J, K] = \int \mathcal{D}\varphi^\alpha \mathcal{D}\pi_\alpha \prod_i \delta(\theta_i)(\det|\{\theta_i, \theta_j\}|)^{1/2} \cdot \exp\left\{i\int d^3x(\pi_\alpha\dot{\varphi}^\alpha - \mathcal{H}_c + J_\alpha\varphi^\alpha + K^\alpha\pi_\alpha)\right\} \tag{13}$$

where φ^α denote all fields, $\varphi^\alpha = (\psi, \overline{\psi}, A_\mu, q)$,and π_α are momenta with respect to φ^α. Here we have introduced the exterior sources J_α with respect to the field φ^α, and the exterior sources K^α with respect to momenta π_α. It is easy to find out that $\det|\{\theta_i, \theta_j\}|$ independent of field variables. Thus, we can omit this factor from the generating functional (13). Using the properties of the δ-function, the expression (13) can be written as (Li and Jiang, 2002)

$$Z[J,K,Y] = \int \mathcal{D}\varphi^\alpha \mathcal{D}\pi_\alpha \mathcal{D}\mu_i \cdot exp\left\{i\int d^3x\left(\mathcal{L}_{\text{eff}}^p + J_\alpha\varphi^\alpha + K^\alpha\pi_\alpha + Y_i\mu_i\right)\right\} \tag{14}$$

where

$$\mathcal{L}_{\text{eff}}^p = \mathcal{L}^p + \mathcal{L}_m \tag{15}$$

$$\mathcal{L}^p = \pi^\mu\dot{A}_\mu + \dot{\psi}\pi_\psi + \dot{\overline{\psi}}\pi_{\overline{\psi}} + \pi_q\dot{q} - \mathcal{H}_c \tag{16}$$

$$\mathcal{L}_m = \mu_i\theta_i \tag{17}$$

μ_i are multiplier fields connected with the constraints θ_i, and Y_i are exterior sources with respect to μ_i.

3. FRACTIONAL SPIN AND STATISTICS

We first formulate the results of the quantal canonical Noether theorem: If the effective action $I_{\text{eff}}^p = \int d^2x\mathcal{L}_{\text{eff}}^p$ is invariant under the following global transformation in extended phase space

$$\begin{cases} x^{\mu'} = x^{\mu} + \Delta x^{\mu} = x\mu + \varepsilon_{\sigma}\tau^{\mu\sigma}(x, \varphi, \pi) \\ \varphi'(x') = \varphi(x) + \Delta\varphi(x) = \varphi(x) + \varepsilon_{\sigma}\xi^{\sigma}(x, \varphi, \pi) \\ \pi'(x') = \pi(x) + \Delta\pi(x) = \pi(x) + \varepsilon_{\sigma}\eta^{\sigma}(x, \varphi, \pi) \end{cases} \tag{18}$$

where φ and π denote: $\varphi = (\psi, q, A_{\mu}, \mu_i)$, $\pi = (\pi_{\psi}, \pi_q, \pi^{\mu})$, respectively, and $\varepsilon^{\sigma}(\sigma = 1, 2, \ldots, r)$ are infinitesimal arbitrary parameters, $\tau^{\mu\sigma}$, ξ^{σ}, η^{σ} are some smoothed functions of canonical variables and time , and if the Jacobian of the transformation (18) of the field variables is equal to unity, then, according to canonical Noether theorem in quantum formalism, there are conserved laws at the quantum level (Li, 1996a,b)

$$Q^{\sigma} = \int d^2x[\pi(\xi^{\sigma} - \varphi_{,k}\tau^{k\sigma}) - \mathscr{H}_{\text{eff}}\tau^{0\sigma}] = \text{const}, \sigma = (1, 2, \ldots, r) \tag{19}$$

where H_{eff} is an effective Hamiltonian density connected with $\mathscr{L}^{p}_{\text{eff}}$. For the Lagrangian (2) whose effective canonical action is invariant under the spatial translation, the Jacobian of the transformation of the field variables is equal to unity, and in this case $\tau^{0\sigma} = 0$. From expression (19), we obtain the conservation of momentum at the quantum level

$$\bar{p} = -\int d^2x(\pi_{\mu}\nabla A_{\mu} + \pi_{\psi}\nabla\psi + \pi_q\nabla q) \tag{20}$$

With the invariance of time translation, $\tau^{i\sigma} = 0\,(i = 1, 2, 3)$, from expression (19), quantal conservation of energy on the constrained hypersurface is given by

$$\begin{aligned} E = \int d^2x \Bigg[& \overline{\psi}(-i\gamma^i D_i + m)\psi + \frac{1}{2\rho}\pi_q^2 + \frac{s}{2}(\nabla q)^2 + G\overline{\psi}\gamma^0\psi q \\ & -\frac{1}{2}sA_i^2q^2 - isA_iq\partial_i q + \frac{1}{2}\rho q^2 A_0^2 \Bigg] \end{aligned} \tag{21}$$

The effective canonical action is invariant under the following global gauge transformation

$$\psi'(x) = e^{-ie\varepsilon}\psi(x), \;\; \pi'_{\psi}(x) = e^{ie\varepsilon}\pi_{\psi}(x)$$

From (19), the conservation of charge at the quantum level is given by

$$Q' = e\int d^2x\overline{\psi}(x)\gamma^0\psi(x) = e\int d^2x\psi^{+}(x)\psi(x) \tag{22}$$

Under the spatial rotation $\tau^{0\sigma} = 0$, and the Jacobian of the transformation of the fields are equal to unity , according to (19), we obtain the quantal conserved angular momentum for this system:

$$L = \int d^2x\varepsilon^{ij}\big[\big(\pi_k S_{ij}^{kl}A_l + x_i\pi^k\partial_j A_k\big) + \big(\pi_{\psi\alpha}R_{ij}^{\alpha\beta}\psi_\beta + x_i\pi_\psi\partial_j\psi\big) + x_i\pi_q\partial_j q\big] \tag{23}$$

where $S_{ij}^{kl} = \delta_i^k\delta_j^l - \delta_j^k\delta_i^l$, $R_{ij}^{\alpha\beta} = \frac{1}{2}(\gamma_i\gamma_j)^{\alpha\beta}$ .The quantal conserved angular momentum under the spatial rotation coincides with the result derived from classical Noether theorem.. Substituting (3b) into (23), using the relations $\varepsilon^{jk}\varepsilon_{il} = \delta_i^j\delta_l^K - \delta_l^j\delta_i^K$, one gets (Bannerjee and Chakraborty, 1994)

$$L = \int d^2x\varepsilon^{ij}x_i(\pi_\psi\partial_j\psi + \pi_q\partial_j q + F^{k0}\partial_j A_k) + \int d^2x\varepsilon^{ij}\pi_{\psi\alpha}R_{ij}^{\alpha\beta}\psi_\beta + \frac{k}{4}\int d^2x[\varepsilon^{ij}x_iA_j(\varepsilon^{lk}\partial_l A_k)] \tag{24}$$

From the Lagrange (2) we write the Euler–Lagrange equation corresponding to A_μ

$$\frac{k}{2}\varepsilon^{\mu\nu\lambda}\partial_\nu A_\lambda = J^\mu \tag{25}$$

let $\mu = 0$, one obtains

$$\frac{k}{2}\varepsilon^{ij}\partial_i A_j = J^0 \tag{26}$$

Observe that (26) leads to a continuity equation

$$\partial_\mu J^\mu = 0 \tag{27}$$

where J^μ is given by

$$J^0 = \overline{\psi}\gamma^0\psi - i\rho(qD^0q), \tag{28}$$

$$J^i = \overline{\psi}\gamma^i\psi + is(qD^iq) \tag{29}$$

If we solve Eq. (26), we obtain (Kim, Kim, and Shin, 1994)

$$A_i(x) = \frac{2}{k}\varepsilon_{ij}\partial_x^j\int d^2yG(x-y)J_0(y) \tag{30}$$

where $G(x-y) = -\frac{1}{2\pi}\ln|x-y| +$ const is the Green's function for the Laplacian in two dimensions, $\partial_i\partial_i G(x,y) = \delta^{(2)}(x-y)$.

Thus, the third term on the right hand of Eq. (24) can be written as

$$\frac{k}{4}\int d^2x\varepsilon^{ij}x_iA_j\varepsilon^{kl}\partial_k Al = \frac{1}{2\pi k}\int d^2xd^2yx_i j^0(x)G(x-y)\partial_i j^0(y) = \frac{1}{2\pi k}\int d^2xd^2yj^0(x)x_k\frac{(x-y)_k}{|x-y|^2}j^0y = \frac{Q^2}{4\pi k} \tag{31}$$

from (24) and (31), one can obtain

$$L = \int d^2x\varepsilon^{ij}x_i(\pi_\psi\partial_j\psi + \pi_q\partial_j q + F^{k0}\partial_j A_k) + \int d^2x\varepsilon^{ij}\pi_{\psi\alpha}R^{\alpha\beta}_{ij}\psi_\beta + \frac{Q^2}{4\pi k} \tag{32}$$

where $Q = \int d^2xJ^0$ (Kim, Kim, and Shin, 1994), the first term on the right-hand of Eq. (32) is the orbit angular momentum operator, the second term is normal spin term operator for the spinor field, the third term is the anomalous one which is interpreted as a spin operator and it is obviously obtained from the orbit angular momentum (Kim, Kim, and Shin, 1994).

Now we denote this spin operator by S, $S = Q^2/4\pi k$, and the one-particle (anyon) state is denoted by $|1>_{\text{any}}$, Then, if one rotates the one-particle state with S, one obtains

$$e^{i\beta s}\left|1>_{\text{any}} = e^{i\beta(1/4\pi k)}\right|1>_{\text{any}} \tag{33}$$

where β is the rotation parameter. The eigenvalue of spin operator S is the spin s. Thus one obtains a relation between the spin s and the CS coefficient k,namely

$$s = \frac{1}{4\pi k} \tag{34}$$

If we take β, as 2π, for $2k = 1/\pi(2n + 1)(n \in Z)$, the one-pratical state picks up a minus sign implying it is a fermionic, and these values of k the spin s take half-integer values. While, for $2k = 1/\pi(2n)(n \in Z)$, the one-practicle state does not change, and hence it becomes bosonic, and the spin s takes integer values, for the other values of k, the state becomes anionic, and the spin s is fractional (Kim, Kim, and Shin, 1994).

4. CHERN–SIMONS TERM COUPLED TO POLARON WITH MAXWELL KINETIC TERM

In (Kim, Kim, and Shin, 1994), the author puts forward that whether the properties of anyons still survive after inclusion of the Maxwell kinetic term or not needs further study. The model with CS term and Maxwell kinetic term coupled to Scalar and spinor QED have been discussed (Jiang and Li, 1999; Li and Li, 2002), in which the fractional spin and statistics still occur at the quantum level. Here we shall study the system with CS term coupled to polaron with Maxwell term.

If we add the Maxwell kinetic term to the Lagrangian (2), in that case the Lagrangian of the system can be written as

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda + \overline{\psi}(i\gamma^\mu D_\mu - m)\psi$$

$$+\frac{1}{2}\{\rho[(\partial_0 - iA_0)q]^2 - s[(\partial_i - iA_i)q]^2\} - G\overline{\psi}\gamma^0\psi q \tag{35}$$

The canonical momenta conjugate to the fields A_μ, ψ, $\overline{\psi}$, and q are

$$\pi^i = -F^{0i} + \frac{k}{4}\varepsilon^{ij}A_j \quad \text{and} \tag{36a}$$

$$\pi^0 = 0, \quad \pi_\psi = i\overline{\psi}\gamma^0, \quad \pi_{\overline{\psi}} = 0, \quad \pi_q = \rho\dot{q}, \tag{36b}$$

respectively. The primary constraints of the system are given by

$$\theta_1^0 = \pi^0 \approx 0, \quad \theta_2^0 = \pi_\psi - i\overline{\psi}\gamma^0 \approx 0, \quad \theta_3^0 = \pi_{\overline{\psi}} \approx 0, \tag{37}$$

The canonical Hamiltonian density corresponding to the Lagrangian is

$$\begin{aligned}\mathcal{H}_c &= \pi_\mu\dot{A}^\mu + \pi_\psi\dot{\psi} + \pi_{\overline{\psi}}\dot{\overline{\psi}} + \pi_q\dot{q} - \mathcal{L} = -\frac{1}{2}\pi^i\pi_i + \frac{k}{2}\varepsilon^{ij}\pi_i A_j \\ &\quad -\frac{k^2}{16}A^iA_j - A_0\partial_i\pi^i + \frac{1}{4}F_{ij}F^{ij} + \overline{\psi}(-i\gamma^iD_i + m)\psi + \frac{1}{2\rho}\pi_q^2 + \frac{s}{2}(\nabla q)^2 \\ &\quad +G\overline{\psi}\gamma^0\psi q - \overline{\psi}\gamma^0A_0\psi - \frac{1}{2}sA_i^2q^2 - isA_iq\partial_iq - \frac{k}{4}\varepsilon^{ij}A_0\partial_iA_j \\ &\quad +\frac{1}{2}\rho A_0^2q^2 + iq\pi_qA_0\end{aligned} \tag{38}$$

The total Hamiltonian is given by

$$H_T = \int d^3x\left(\mathcal{H}_c + \lambda_1\theta_1^0 + \lambda_2\theta_2^0 + \lambda_3\theta_3^0\right) \tag{39}$$

The consistency condition $\{\theta_1^0, H_T\} \approx 0$ leads to secondary constraints

$$\theta_1^1 = \partial_i\pi^i + \frac{k}{4}\varepsilon^{ij}\partial_iA_j - \rho q^2A_0 - iq\pi_q + \overline{\psi}\gamma^0\psi \approx 0 \tag{40}$$

The consistency conditions of the primary constraints $\dot{\theta}_2^0 \approx 0$, $\dot{\theta}_3^0 \approx 0$ and the secondary constraints $\dot{\theta}_1^1 \approx 0$, lead to the equation for determining the Lagrange multiplier $\lambda_3, \lambda_2, \lambda_1$, but do not produce additional constraints. It is easy to check that the constraints are all second-class constraints.

Using the quantal canonical Noether theorem, one can proceed in the same way as before to obtain the quantal conserved quantities.The conservation of momentum at the quantum level which is similar to the result without inclusion of Maxwell kinetic term is

$$\bar{P} = -\int d^2x(\pi_\psi\nabla\psi + \pi_q\nabla q + \pi^\mu A_\mu) \tag{41}$$

The conservation of energy on the constrained hypersurface at the quantum level is

$$E = \int d^2x \left[-\frac{1}{2}\pi^i \pi_i + \frac{k}{2}\varepsilon^{ij}\pi_i A_j - \frac{k^2}{16}A^i A_j + \frac{1}{4}F_{ij}F^{ij} + \overline{\psi}(-i\gamma^i D_i + m)\psi \right.$$
$$\left. + \frac{1}{2\rho}\pi_q^2 + \frac{s}{2}(\nabla q)^2 + G\overline{\psi}\gamma^0\psi q - \frac{1}{2}sA_i^2 q^2 - isA_i q\partial_i q - \frac{1}{2}\rho A_0^2 q^0 \right] \quad (42)$$

The quantal conserved angular momentum for this system is

$$L = \int d^2x\varepsilon^{ij}\left(\pi_k S_{kl}^{ij} A_l + x_i \pi^k \partial_j A_k\right) + \left(\pi_{\psi\alpha} R_{ij}^{\alpha\beta}\psi_\beta + x_i \pi_\psi \partial_j \psi\right) + x_i \pi_q \partial_j q \quad (43)$$

Substituting (36a) into (43) ,one gets

$$L = \int d^2x\varepsilon^{ij} x_i (F^{k0}\partial_j A_k + \pi_\psi \partial_j \psi + \pi_q \partial_j q) + \int d^2x\varepsilon^{ij}\pi_{\psi\alpha} R_{ij}^{\alpha\beta}\psi_\beta$$
$$+ \int d^2x\varepsilon^{ij} F_{ko} S_{ij}^{kl} A_l + \int d^2x\varepsilon^{ij}\frac{k}{4}x_i A_j(\varepsilon^{lk}\partial_l A_k) \quad (44)$$

From the Lagrange (35) the Euler–Lagrange equation corresponding to $A_\mu$is given by

$$\partial_\nu F^{\mu\nu} + \frac{k}{2}\varepsilon^{\mu\nu\lambda}\partial_\nu A_\lambda + J^\mu = 0 \quad (45)$$

let $\mu = 0$, one obtains

$$\partial_i \pi^i + \frac{k}{4}\varepsilon^{ij}\partial_i A_j + J^0 = 0 \quad (46)$$

from (46), we can obtain

$$A_i(x) = -\frac{4}{k}\varepsilon_{ij}\partial_x^j \int d^2y G(x-y) J^0(y) \quad (47)$$

From (47), (44) becomes

$$L = \int d^2x\varepsilon^{ij} x_i (F^{k0}\partial_j A_k + \pi_\psi \partial_j \psi + \pi_q \partial_j q) + \int d^2x\varepsilon^{ij}\pi_{\psi\alpha} R_{ij}^{\alpha\beta}\psi_\beta$$
$$+ \int d^2x\varepsilon^{ij} F_{k0} S_{ij}^{kl} A_l - \frac{Q^2}{\pi k} \quad (48)$$

where $Q = \int d^2x J^0$. It can be seen from (48) that the Maxwell term contributes the orbit and spin terms to the total angular momentum. The fractional spin property is still presented but the result is different from the one that is derived of the model without the inclusion of Maxwell term.

5. CONCLUSION AND DISCUSSION

We have treated the model with Abelian CS term coupled to polaron without Maxwell kinetic terms and with it separately at the quantum level. According to the rule of path integral quantization for constrained Hamiltonian system in FS scheme the system is quantized. Based on the quantal Noether theorem, we derived a generalized spin-statistics relation at the quantum level by computing the angular momentum. The Maxwell kinetic term does not alter the property of fractional spin.

In this case, all constraints are second-class constraints, so there are no gauge transformations corresponding to first-class constraints. But it will be convenient to discuss the same problem by Dirac canonical quantization. It needs further study to determine whether the fractional spin can be described by the formalism of quantization or not.

It needs further study to see whether the property of fractional spin is still presented or not at the quantum level after including the non-Abelian CS term.

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